2017/10/13 (Fri)
東大 cp seminar

time ① parameter, ② dinamics, ③ q observable.
POV, CCR

$[P, Q] = -i$

$[H, T] = -i \qquad T = i\frac{d}{dH}$

$e^{-isH} e^{-itT} = e^{ist} e^{-itT} e^{-isH}$ WR — von Neumann

$T e^{-itH} = e^{-itH}(T+t)$ WWR

$[H, T] = -i$ CCR

$(H\varphi, T\psi) - (T\varphi, H\psi) = -i(\varphi, \psi)$ w-CCR

$t[H\varphi, \psi] - t[\varphi, H\psi] = -i(\varphi, \psi)$ q-CCR

WR - WWR - CCR - wCCR - q CCR
us s time w uw

$H = H_{ac} \oplus H_{sc} \oplus H_p \qquad H = -\frac{1}{2}\Delta + V$ in $L^2(\mathbb{R}^d) = \mathcal{H}$

$\boxed{H_{ac}}$ WWR $\to \sigma(H) = \sigma_{ac}(H)$, $H > -\infty \Rightarrow T$ s.a. をもたない

Example: $\frac{1}{2}\sum(Q_j \frac{1}{P_j} + \frac{1}{P_j} Q_j) = T_{AB}$

$[H, T] = -i \Rightarrow [f(H), {}^{\exists}T_f] = -i$?

$[T, f(H)] = if' \quad \therefore [\frac{1}{f'}T, f] = i \quad \therefore \frac{1}{2}\left(\frac{1}{f'}T + T\frac{1}{f'}\right)$

Lemma $\exists W_\pm = \lim_{t\to\pm} e^{itH} e^{-itH_0}$, $\operatorname{Ran}(W_\pm) = \mathcal{H}$

$\Rightarrow W_\pm T_{AB} W_\pm^{-1} = T_{ac}$ は H_{ac} と WWR.

☺ $T_{ac} e^{-itH} = W T_{AB} e^{-itH_0} W^{-1} = W e^{-itH_0}(T_{AB}+t) W^{-1}$ //

$\boxed{H_p}$ Galapon, Arai-Matsuzawa

一般論 $\sigma(H) = \{E_j\}$ ① simple ② $E_j \to \infty$ ③ $\sum \frac{1}{E_j^2} < \infty$

$$T\phi = i\sum_{n=1}^{\infty}\left(\sum_{m\neq n}\frac{(e_m, \phi)}{E_n - E_m}\right)e_n \Bigg/ \left|\sum\frac{(e_m,\phi)}{E_n - E_m}\right| \le \left(\sum\frac{1}{|E_n - E_m|^2}\right)^{1/2}\left(\sum|(e_m,\phi)|^2\right)^{1/2} < \infty$$

$\therefore [H, T] = i$ on $\mathcal{E} = \{e_n - e_m\}$

Remark. $e_n \notin D(T)$ but $e_n - e_m \in D(H)$ ✓ (A)

Lemma $\sigma(H) = \{E_j\}$ $E_j \to 0$, $\sum E_j^2 < \infty$, $0 \notin \sigma_p(H)$.

$\Rightarrow H^{-1}$ の WWR.

∵ $\sigma(H^{-1}) = \{\frac{1}{E_j}\}$ $\frac{1}{E_j} \to \infty$, $\sum E_j^2 < \infty$ //

Lemma 上と同じとき $\exists t$, $\mathcal{E} = H^{-1}\{e_n - e_m\}$. で CCR

∵ $H = f(H^{-1})$ $f(x) = x^{-1}$

$T = \frac{1}{2}\left(\frac{1}{f'}T + T\frac{1}{f'}\right) = -\frac{1}{2}\left(H^{-2}T + TH^{-2}\right)$

$t[\varphi, \psi] = -\frac{1}{2}[(T\varphi, H^{-2}\psi) + (H^{-2}\varphi, T\psi)]$

$\varphi, \psi \in H^{-1}\mathcal{E}$.

Thm Arai - H. $\sigma(H) = \{E_j\}$ ① $\lim E_j = 0$ ② $0 \notin \sigma_p$.

$\exists t$ で CCR

∵ $H = \oplus H_j$ H_j は (A) をみたす. //

Thm $H \in S^1 \iff$ ① $\sigma_{sc}(H) = \emptyset$
② $\sigma_{ac}(H) = [0, \infty)$ $\exists T_{ac}$
③ $\sigma_{dis}(H) = \{E_j\}$ $E_j \to 0$
$0 \notin \sigma_p(H)$

$\Rightarrow \exists t$ で - CCR.

∵ $H = H_{ac} \oplus H_p$

$\tau_H(\varphi, \psi) = (\psi, T_{ac}\varphi) + \tau_p(\varphi, \psi)$ //

Ex. $d=3$ $V(x) = \dfrac{W(x)}{(1+|x|^2)^{\frac{1}{2}+\varepsilon}}$ ① $W \le 0$, $W(x) = -\frac{1}{|x|^\alpha}$ $0 < \alpha < 1$, $|x| > R$

③ $2\varepsilon + \alpha < 1$.

w spherical, cont.

$\frac{1}{|x|^2}$ が critical

Ex $d=3$ H_{hyd}.